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Title: Uncomputability of some class of quantum control problems

Abstract:

Uncomputable in the Turing sense problems do sometimes appear in mathematics and quantum physics. In mathematics, famous examples include Halting problem and Hilbert's 10 problem about existence of solutions of Diophantine equations. In quantum physics, examples include undecidability of the spectral gap and ground state of a boson Hamiltonian [1,2,3]. In this talk, we will discuss obtained with D.I. Bondar result about uncomputability of determining the existence of globally optimal solutions for a discrete version of quantum control [4]. Consider a controlled open quantum system with density matrix ρ. The system can be controlled using coherent (e.g., a laser field) [5] or incoherent (e.g., spectral density of the environment) [6,7] controls. Most general controlled evolution of density matrix of the system is represented by a Kraus map, i.e., a completely positive trace preserving linear map [8]. Suppose we have access to N elementary controls (e.g., N laser pulses of different intensities) which we can combine in an arbitrary order or in some set of accessible sequences. These N elementary controls induce N Kraus maps ϕ_i, i=1… N, which represent evolutions of the system. Control pulse consisting of L elementary controls is defined by a sequence $p=(p_1,\dots,p_L)$. It determines the map $\phi_p = \phi_{pL} \dots \phi_{p1}$ which acts on the system's initial state ρ_0. Consider control problem of maximizing average value of an observable O (without loss of generality one can assume that this maximum is 0). We show that there is no algorithm which can decide, given as input ρ_0, O, $\{\phi_i\}$ (they all can be defined with some precision by rational numbers), whether there exists a control sequence p^* which provides global maximum of O (i.e., zero value of its average in the state induced from the initial state by the control sequence). We establish a connection between discrete quantum control problems and Diophantine equations based on which we prove that there is no such algorithm [4]. Moreover, our method allows to transfer to quantum control known results on complexity of solving various Diophantine equations thereby establishing discrete quantum control problems of various complexity (e.g., constructing NP-hard quantum control problems) [4]. This research for open quantum systems was supported by the Russian Science Foundations Project 17-11-01388 and for closed quantum systems by the project 1.669.2016/1.4 of the Ministry of Science and Higher Education of the Russian Federation.

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