# On Time-Minimal Control Problems, Reachable Sets, and Machine Learning Control for an Open Two-Level Quantum System

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Optimal control theory is an important tool for controlling quantum systems [1]. Based on the article [2] considering both coherent and incoherent controls in the Gorini–Kossakowski–Sudarshan–Lindblad master equation for Markov open quantum dynamics, the two-level case with some certain Hamiltonian was considered in the recent articles [3 – 5] in the context of analyzing time-minimal control, reachable and controllability sets of the system. The talk is devoted to some part of the results obtained for this open quantum system in the papers [4, 5]. The exact analytical results on the system's reachability sets and time-minimal control correspond to the case of coherent control being zero all the time and incoherent control varying in certain class of constant functions [4]. For some class of time-minimal control problems, the regression problem was formulated [5], where the learning results were found by applying numerical optimization (dual annealing and differential evolution methods). For the regression problem, some approach with the kNN method and neural networks was composed. The corresponding suboptimal results were found for different numbers of the training samples.

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## References

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