Nonlinear map in probability representation as purification method of qubit states

Ivan Dudinets
Vladimir Man'ko

September 11, 2018

Moscow Institute of Physics and Technology

MIPT (PhysTech)-QUANT-2018

ontents

Parametrizations

density matrix

$$\rho^\dagger = \rho\,, \quad \rho \geq 0\,, \quad \mathrm{Tr}\rho = 1$$

parametrization of density matrices

- Bloch parametrization

[1] K. Zyczkowski and H.-J. Sommers, J. Phys. A: Math. and Gen, **34** (2001) 7111
[2] N Il'in, et al., J. Phys. A: Math. and Theor, **51** (8) (2018) 085301
[3] V. N. Chernega, V. I. Man'ko, et al., JRLR, **38** (2017) 141

parametrization of density matrices

- Bloch parametrization
- Squaring parametrization [1] $\rho = AA^\dagger / \mathrm{Tr} AA^\dagger$

[1] K. Zyczkowski and H.-J. Sommers, J. Phys. A: Math. and Gen, **34** (2001) 7111
[2] N Il'in, et al., J. Phys. A: Math. and Theor, **51** (8) (2018) 085301
[3] V. N. Chernega, V. I. Man'ko, et al., JRLR, **38** (2017) 141

parametrization of density matrices

- Bloch parametrization
- Squaring parametrization [1] $\rho = AA^\dagger / \mathrm{Tr} AA^\dagger$

  [2] $\rho = B^2 / \mathrm{Tr} B^2$

---

[1]K. Zyczkowski and H.-J. Sommers, J. Phys. A: Math. and Gen, **34** (2001) 7111

[2]N Il'in, et al., J. Phys. A: Math. and Theor, **51** (8) (2018) 085301

[3]V. N. Chernega, V. I. Man'ko, et al., JRLR, **38** (2017) 141

parametrization of density matrices

- Bloch parametrization
- Squaring parametrization [1] $\rho = AA^\dagger / \mathrm{Tr} AA^\dagger$
  [2] $\rho = B^2 / \mathrm{Tr} B^2$
- Probability parametrization [3]

[1] K. Zyczkowski and H.-J. Sommers, J. Phys. A: Math. and Gen, **34** (2001) 7111
[2] N Il'in, et al., J. Phys. A: Math. and Theor, **51** (8) (2018) 085301
[3] V. N. Chernega, V. I. Man'ko, et al., JRLR, **38** (2017) 141

tomographic probability representation

[1] $$\rho \leftrightarrow w(m,\vec{n}) = \langle m|u(\vec{n})\rho u^\dagger(\vec{n})|m\rangle$$

$$\vec{j}^2|m\rangle = j(j+1)|m\rangle\,, \quad j_z|m\rangle = m|m\rangle$$

$$m = -j\ldots,j$$

[1] Dodonov V. and Manko V., PLA, **229** (1997) 335.
Man'ko V. and Man'ko O., JETP, **85** (1997) 430.

properties of the spin tomogram

- nonnegativity $w(m, \vec{n}) \geq 0$
- normalization $\sum_{m=-j}^{j} w(m, \vec{n}) = 1$

spin-j system

redundant information

$w(m, \vec{n})$	m $2j+1$ values $\vec{n}$ continuous parametrization
ρ	$(2j+1)^2 - 1$ real parameters

avoiding redundancy

$2j+2$ directions $\vec{n}_k$

for every $\vec{n}_k$ we have $2j$ probabilities $w(m,\vec{n})$

altogether $\quad 2j(2j+2)=(2j+1)^2-1$

spin-1/2 system

$2j + 2 = 3$ directions

$$p_1 = w(m = 1/2, \vec{x}) \,, \quad p_2 = w(m = 1/2, \vec{y}) \,, \quad p_3 = w(m = 1/2, \vec{z})$$

spin-1/2 system

$2j + 2 = 3$ directions

$$p_1 = w(m = 1/2, \vec{x}), \quad p_2 = w(m = 1/2, \vec{y}), \quad p_3 = w(m = 1/2, \vec{z})$$

$$\rho = \frac{I}{2} + \left(p_k - \frac{1}{2}\right) \sigma_k$$

probability parametrization

$$\rho = \frac{I}{2} + \left(p_k - \frac{1}{2}\right)\sigma_k$$

$$(p_1 - 1/2)^2 + (p_2 - 1/2)^2 + (p_3 - 1/2)^2 \leq 1/4$$

Nonlinear map

nonlinear map

[1] $$\rho \to \Phi_\alpha(\rho) = \frac{\rho^\alpha}{\mathrm{Tr}\rho^\alpha}$$

[1] V. I. Manko and R. S. Puzko, JRLR, **35** (2014) 362.
V. I. Manko and R. Puzko, EPL,**109** (2015) 50005.

escort distribution

$$^{1} \quad p_i \rightarrow P_i = p_i^{\alpha} / \sum_j p_j^{\alpha}$$

[1]C. Beck and F. Schlögl, Thermodynamics of Chaotic Systems. Vol. 4, Cambridge Univ. Press 294 (UK) 1993.

thermal state

$$\rho = Z^{-1} \exp(-\beta H)$$

$$Z = \operatorname{Tr} \exp(-\beta H)$$

$$\rho \to \Phi_\alpha(\rho) \Rightarrow \beta \to \alpha\beta$$

nonlinear map in probability parametrization

[1] $$\Phi_\alpha(\rho) = \frac{I}{2} + \left(\tilde{p}_k - \frac{1}{2}\right)\sigma_k$$

$$\tilde{p}_k - \frac{1}{2} = \frac{\left(p_k - \frac{1}{2}\right)\sqrt{\mu_\alpha - \frac{1}{2}}}{\sqrt{\left(p_1 - \frac{1}{2}\right)^2 + \left(p_2 - \frac{1}{2}\right)^2 + \left(p_3 - \frac{1}{2}\right)^2}}$$

$$\mu_\alpha = \mathrm{Tr}\left(\Phi_\alpha(\rho)\right)^2 = \frac{\lambda_+^{2\alpha} + \lambda_-^{2\alpha}}{(\lambda_+^\alpha + \lambda_-^\alpha)^2}$$

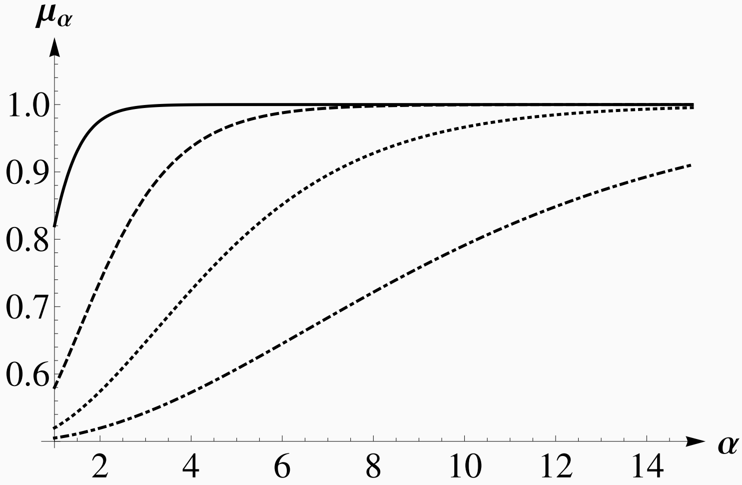
[1] I. V. Dudinets and V. I. Man'ko, EPL

purity parameter

$$\mu = \mathsf{Tr}\rho^2$$

$$0.5 \leq \mu \leq 1$$

$\mu = 0.5$	ρ=I/2
$\mu = 1$	$\rho = \|\psi\rangle\langle\psi\|$



The purity μ_α for $\lambda_+ = 0.9$ (solid line), $\lambda_+ = 0.7$ (dashed line), $\lambda_+ = 0.6$ (dotted line), $\lambda_+ = 0.55$ (dot-dashed line), versus the parameter of the nonlinear map α.

Distance between states

Tsallis relative entropy

$$^{1}\quad S_q(\rho \,\|\, \sigma) = \mathrm{Tr}\left(\rho^q \ln_q \rho - \rho^q \ln_q \sigma\right)$$

q-logarithmic function

$$\ln_q x = (x^{1-q} - 1)/(1-q)$$

[1]C. Tsallis, Phys. Rev. E, **58** (1998) 1442.
L. Borland, A. R. Plastino and C. Tsallis, J. Math. Phys., **39** (1998) 6490.
M. Shiino, J. Phys. Soc. Japan, **67**, (1998) 3658

Tsallis relative entropy

[1] $$S_q(\rho \parallel \sigma) = \mathrm{Tr}\left(\rho^q \ln_q \rho - \rho^q \ln_q \sigma\right)$$

q-logarithmic function

$$\ln_q x = (x^{1-q} - 1)/(1-q)$$

$$q \to 1 \qquad S_q(\rho \parallel \sigma) \to S(\rho \parallel \sigma) = \mathrm{Tr}\left(\rho \ln \rho - \rho \ln \sigma\right)$$

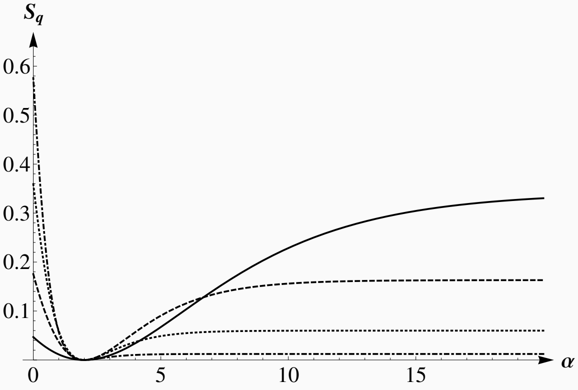
[1] C. Tsallis, Phys. Rev. E, **58** (1998) 1442.
L. Borland, A. R. Plastino and C. Tsallis, J. Math. Phys., **39** (1998) 6490.
M. Shiino, J. Phys. Soc. Japan, **67**, (1998) 3658

Tsallis relative entropy

$$S_q(\rho \parallel \sigma) = \frac{1}{1-q}\left(1 - \mathrm{Tr}\rho^q \sigma^{1-q}\right)$$

entropy for $\Phi_\alpha(\rho)$ and $\Phi_\beta(\rho)$

$$S_q(\Phi_\alpha(\rho) \parallel \Phi_\beta(\rho)) = \frac{1}{1-q}\left(1 - \frac{\lambda_+^{\alpha q+\beta(1-q)} + \lambda_-^{\alpha q+\beta(1-q)}}{\left(\lambda_+^\alpha + \lambda_-^\alpha\right)^q \left(\lambda_+^\beta + \lambda_-^\beta\right)^{1-q}}\right)$$



The relative Tsallis entropy S_q for $\lambda_+ = 0.6$ (solid line), $\lambda_+ = 0.7$ (dashed line), $\lambda_+ = 0.8$ (dotted line), $\lambda_+ = 0.9$ (dot-dashed line), versus α. Parameters $\beta = 2$ and $q = 0.6$.

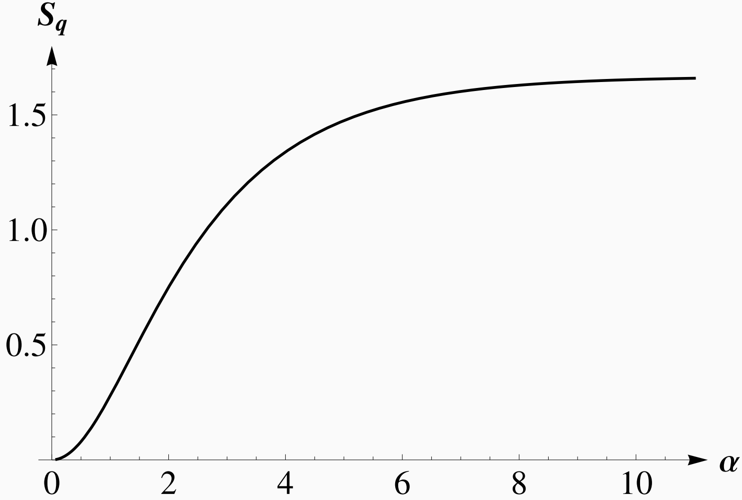
entropy for $\Phi_\alpha(\rho_1)$ and $\Phi_\alpha(\rho_2)$

$$\rho_1 = \begin{pmatrix} 0.7 & 0 \\ 0 & 0.3 \end{pmatrix}, \quad \rho_2 = \begin{pmatrix} 0.5 & 0.25(1+I) \\ 0.25(1-I) & 0.5 \end{pmatrix}$$

$$S_q(\Phi_\alpha(\rho_1) \parallel \Phi_\alpha(\rho_2)) = \frac{1}{1-q}\left(1 - \frac{(\lambda_1^{\alpha q} + (1-\lambda_1)^{\alpha q})\left(\lambda_2^{\alpha(1-q)} + (1-\lambda_2)^{\alpha(1-q)}\right)}{2\left(\lambda_1^{\alpha} + (1-\lambda_1)^{\alpha}\right)^q \left(\lambda_2^{\alpha} + (1-\lambda_2)^{\alpha}\right)^{1-q}}\right)$$

$$\lambda_1 = 0.7 \qquad \lambda_2 = 0.5\left(1 + 2^{-1/2}\right)$$

entropy for $\Phi_\alpha(\rho_1)$ and $\Phi_\alpha(\rho_2)$



The relative Tsallis entropy S_q versus α. Parameter $q = 0.7$.

Conclusion

Summary

- the state of a spin-1/2 state can be represented by means of the probabilities to have the spin projection $m = 1/2$ on the three orthogonal axes in space.
- the nonlinear map for large values of its parameter gives the density matrix corresponding to either the maximally mixed or pure state
- we studied the distinguishability between transformed states with the help of the Tsallis relative entropy

Thank you